Static, kinematic, dynamic, and generic invariants in touch

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wearable haptics for Humans and Robots

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What is an invariant? Examples:

- The fixed points of a transformation, f, are those points, x, such that $f(x) = x$, and thus are invariants for that transformation.
- $ax^2 + bxy + cy^2 + dz + ey + f = 0$
  The discriminant, $b^2 - 4ac$, which tells the type of graph the equation traces is invariant under rotations.*
- $ax + by + c = 0, \quad \forall\alpha : x = x' + \alpha,\ y = y' - \frac{a}{b}\alpha$, transforms the line into to the same line.

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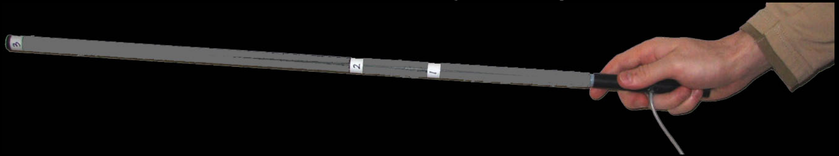
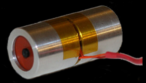
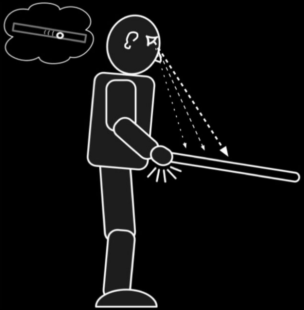
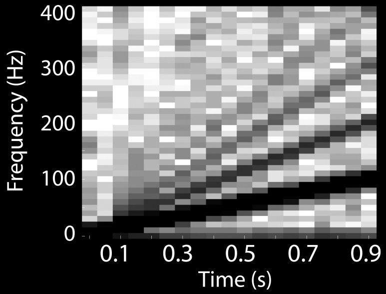
* Apollonius of Perga (c. 200 BC) Treatise on Conic Sections.

Some invariants are in the ambient physics, e.g.









Yao, H.-Y. and Hayward, V. 2006. An Experiment on Length Perception with a Virtual Rolling Stone. Proc. *Eurohaptics 2006*. pp. 325-330.

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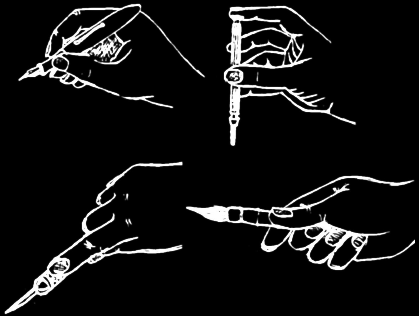
Some others are in the sensing apparatus

invariants, and priors

indirect touch:

$$g \circ f \overset{?}{\Leftrightarrow} f^{-1} g^{-1}$$

f: object-tool transformation
g: transformation tool-hand transformation



direct contact is easier for perception
therefore more difficult to stimulate

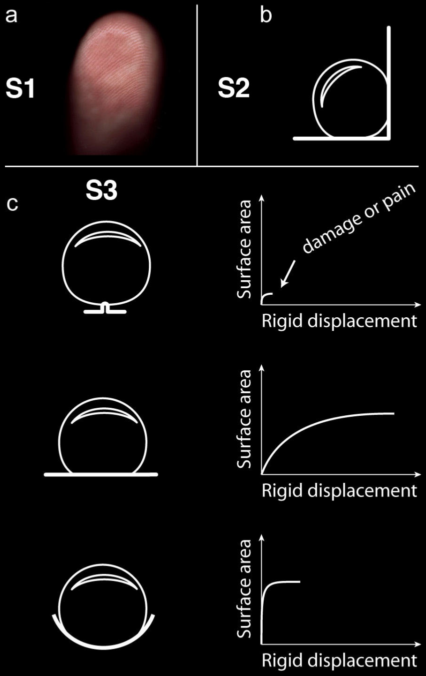
Hayward, V. 2008. Haptic Shape Cues, Invariants, Priors, and Interface Design. In "Human Haptic Perception - Basics and Applications", Grunwald, M. (ed.), Birkhauser Verlag, pp. 381–392.

static invariants

S1 = biological metric

S2 (and others) = geometry

S3 = contact mechanics



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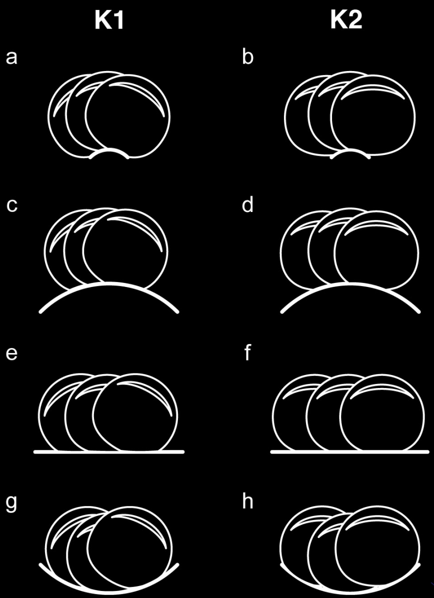
kinematic invariants

(principe de St Venant)

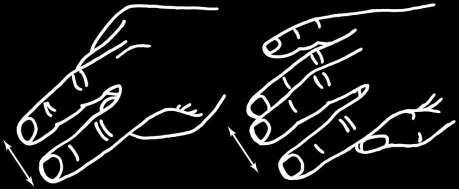
$$\text{K1}: \; k \in]\infty, -k_d[\; \mapsto \frac{\mathrm{d}x_c}{\mathrm{d}\theta_d} \in]0, \infty[$$

$$\text{K2}: \; k \in]\infty, -k_d[\; \mapsto \frac{\mathrm{d}x_c}{\mathrm{d}x_d} \in]0, \infty[$$

k : object curvature
k_d : finger curvature
x_c : somatotopic location
θ_d : finger orientation in space
x_d : finger position in space







K1

K2

flatness detection using K2

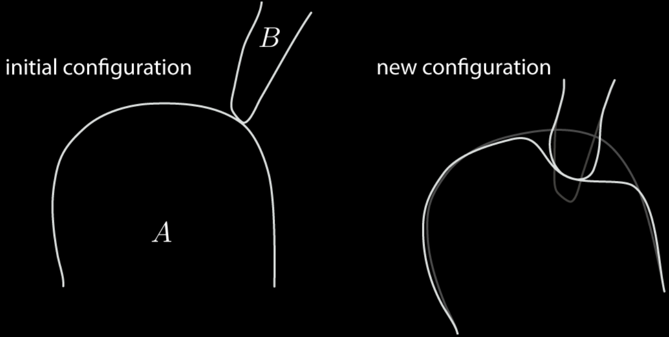


the 'plenhaptic' function

- with vison, $p(l, v, \lambda, t)$ is in $\mathbb{R}^7$

  Adelson E. H., Bergen J. R. '81. The **plenoptic** function and the elements of early vision

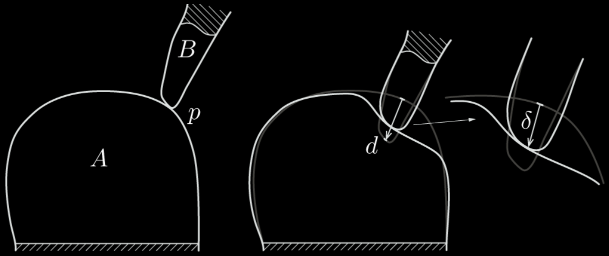
- with touch,



$$a = h_{A,B}(b) \quad b = h_{B,A}(a)$$

Hayward, V. 2011. Is There a 'Plenhaptic' Function? *Philosophical Transactions of the Royal Society B*, 366:3115–3122

simplifications



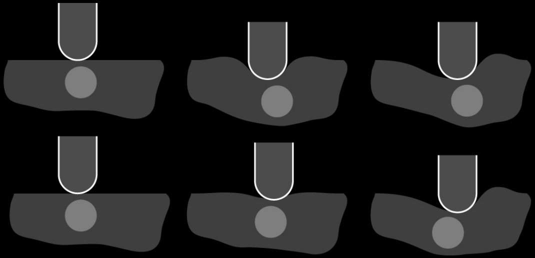
$$\delta = \bar{h}_{A,B}(p, d)$$

ou $\quad d(t) : \mathbb{R}^+ \mapsto \mathbb{R}^3 \times SO(3)$

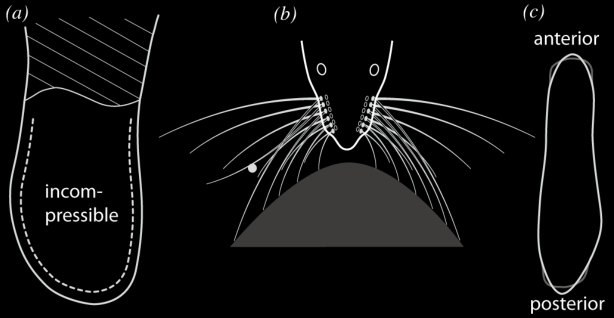
local deformation assumption

simplifications

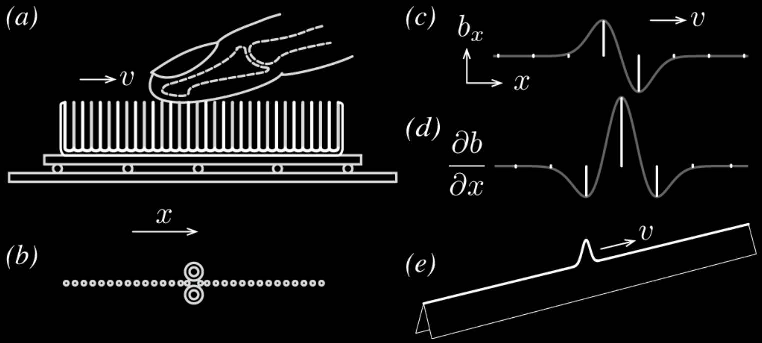
- rigid objects and rigid probe: satisfy $0 \approx \bar{h}(p, d)$
- rigid objects et soft probe: find object, B, such that $0 = h(b)$
- soft objects et rigid probe: $\delta = h(p, d)$ or $h(p, d) = d - p$
- complexities:



haptic probes

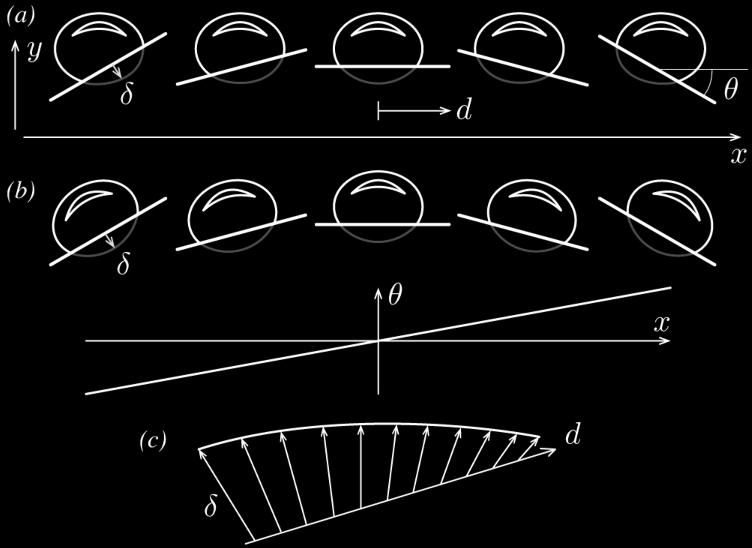


tactiles illusions



Hayward, V., Cruz-Hernandez, M. 2000. Tactile Display Device Using Distributed Lateral Skin Stretch. Proc. *Haptic Interfaces for Virtual Environments and Teleoperator Systems Symposium*, ASME Vol. DSC-69-2, pp. 1309-1314

haptic illusions



Dostmohamed, H., Hayward. V., 2005. Trajectory of Contact Region On the Fingerpad Gives the Illusion of Haptic Shape. *Experimental Brain Research*. 164:387-394.